Killara HIGH SCHOOL

MATHEMATICS DEPARTMENT

Student Number

2023 **Year 12 Trial Examination**

Mathematics Advanced

07/08/2023

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using blue or black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- No white-out may be used

Total Marks: 100

Section I - 10 marks (pages 3–10)

- Allow about 15 minutes for this section

Section II - 90 marks (pages 11–44)

- Allow about 2 hours and 45 minutes for this section

This question paper must not be removed from the examination room.

This assessment task constitutes 40% of the course.

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section.

Use the multiple-choice sheet for Question 1–10.

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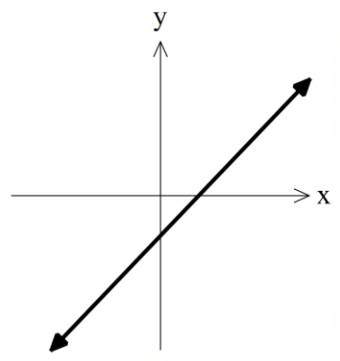
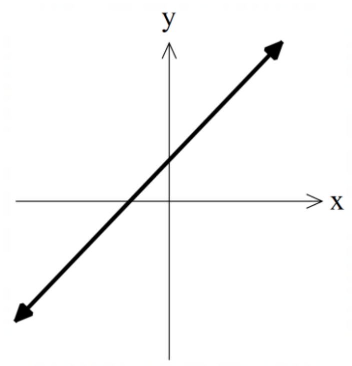
1 Which one of the following could be $y = -x + 3$?

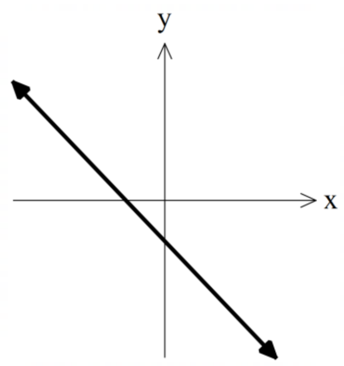
A.



B.



C.



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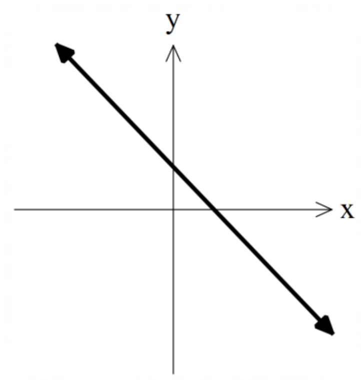


2 The probability distribution of the random variable X is given in the table below. Calculate $E(X)$

x	2	3	4	5
$P(X = x)$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$	

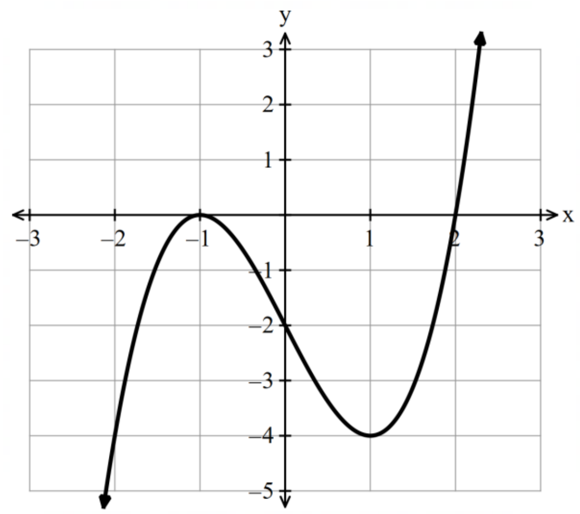
A. $\frac{1}{6}$

B. 2

C. $\frac{8}{3}$

D. $\frac{7}{2}$

3 Which of the following equations best describes the graph below?



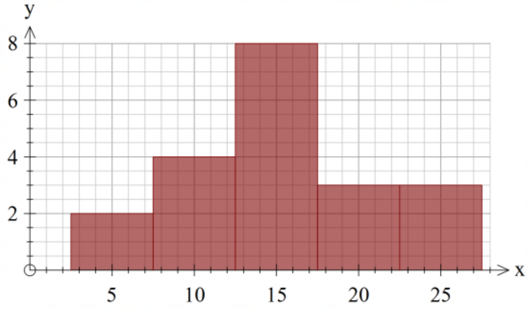
A. $y = (x + 1)^2(x - 2)$

B. $y = (x + 1)(x - 2)^2$

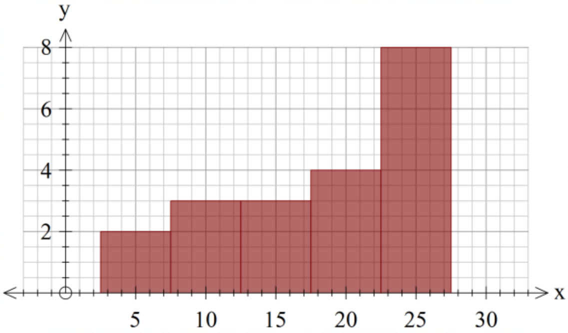
C. $y = (x - 1)^2(x + 2)$

D. $y = (x - 1)(x + 2)^2$

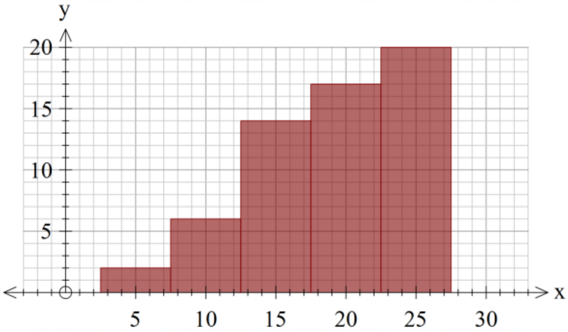
4 A frequency histogram is given below where the y-axis represents the frequency. Which one of the following is the corresponding cumulative frequency histogram?



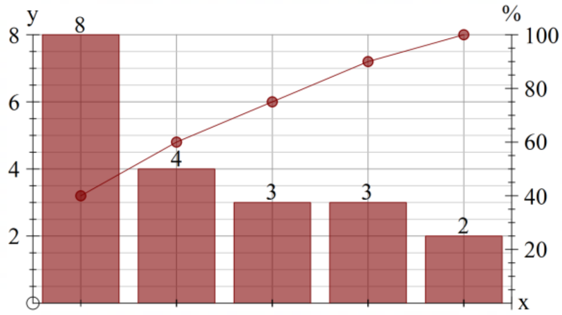
A.



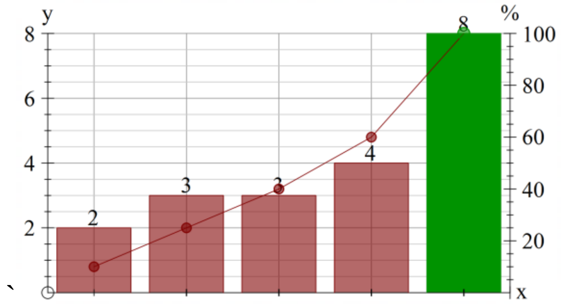
B.



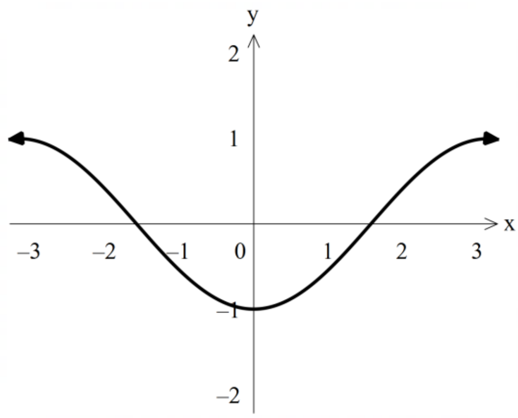
C.



D.

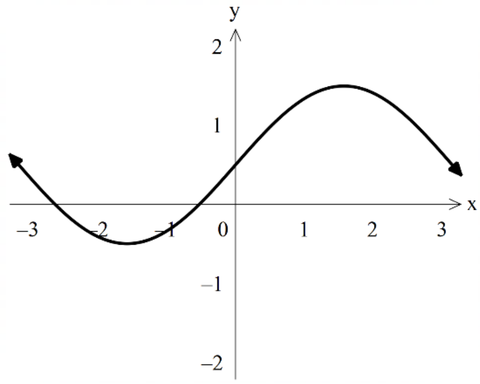


5 The gradient function, $f'(x)$ is sketched below.

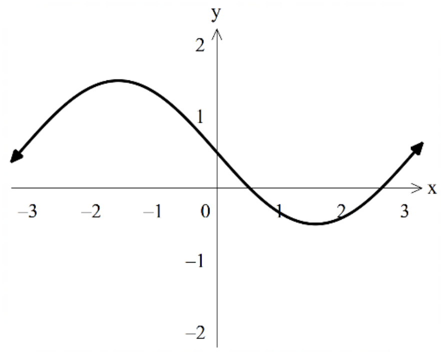


Which one is the possible graph of $f(x)$?

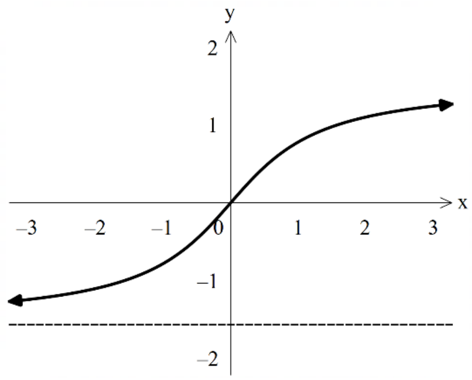
A.



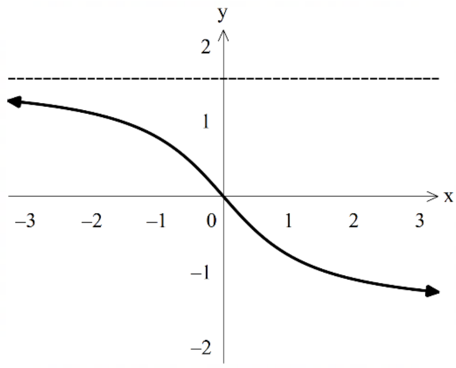
B.



C.



D.



6 A function is defined as:

$$f(x) = x^2 - 6x + 10 \text{ for } x \in [-1, 3).$$

What is its range?

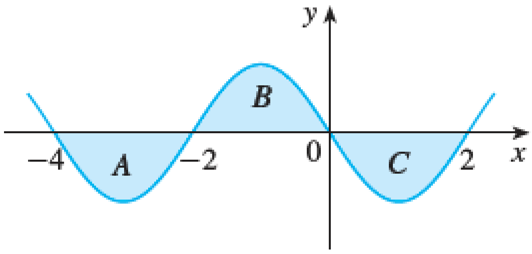
A. $y \in (1, \infty)$

B. $y \in [1, \infty)$

C. $y \in (1, 17]$

D. $y \in [1, 17]$

7 Each of the regions A, B and C bound by the graph of f and the x-axis has the same area. If $\int_0^2 [f(x) + 2x + 5]dx = 11$, what is the value of $\int_{-4}^{2} f(x)dx$



A. $\int_{-4}^{2} f(x)dx = -6$

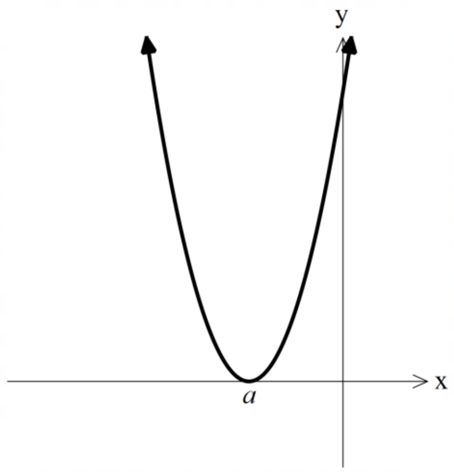
B. $\int_{-4}^{2} f(x)dx = 6$

C. $\int_{-4}^{2} f(x)dx = -3$

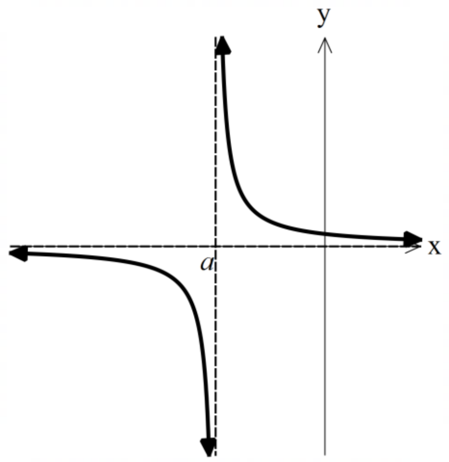
D. $\int_{-4}^{2} f(x)dx = 3$

8. The graphs of $f(x)$ and $g(x)$ are shown below. It is known that the turning point of $f(x)$ occurs where $x = a$ and the vertical asymptote of $g(x)$ is $x = a$.

$\boldsymbol{y = f(x)}$

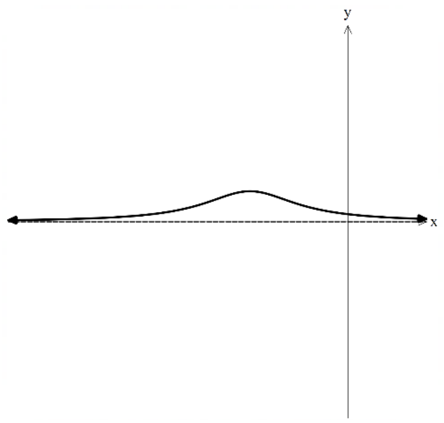


$\boldsymbol{y = g(x)}$



Which one of the following graphs best represents the graph of $g(f(x))$?

A.



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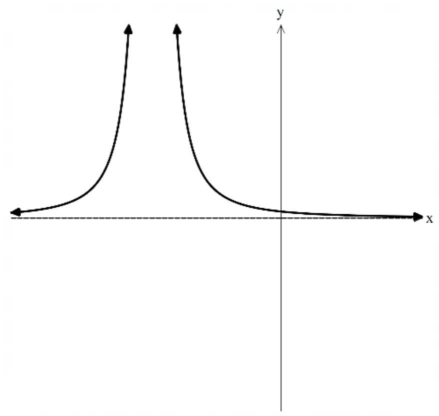

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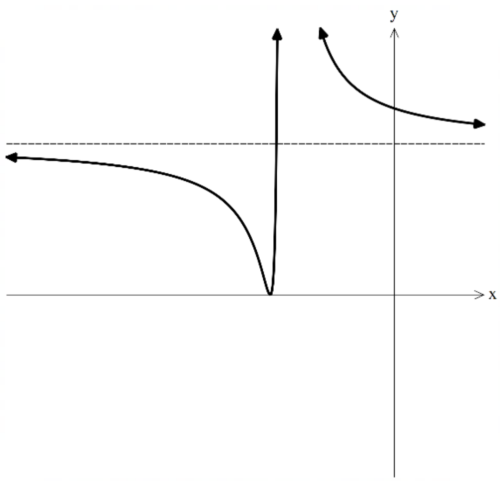
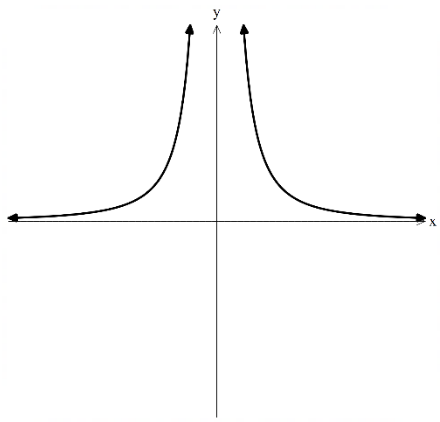


D.



9 What is $\int \frac{1}{1-x} dx$?

A. $\ln|1-x| + c$

B. $\ln\left|\frac{1}{1-x}\right| + c$

C. $\ln 1 - \ln|x| + c$

D. $-\ln|x| + c$

10 A probability density function is given below:

$$f(x) = \begin{cases} x-1, & 1 \le x \le a \\ 0, & else\ where \end{cases}$$

Which one of the following is the correct way to find the value of a

A. The larger root of $a^2 - 2a - 1 = 0$

B. The smaller root of $a^2 - 2a - 1 = 0$

C. The larger root of $a^2 + 2a - 1 = 0$

D. The smaller root of $a^2 + 2a - 1 = 0$

End of Section I

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Killara HIGH SCHOOL
MATHEMATICS DEPARTMENT

Student Number

2023 **Year 12 Trial Examination**

Mathematics Advanced

Section II Answer Booklet 1

07/08/2023

Section II

90 Marks
Attempt Questions 11–37
Allow about 2 hours and 45 minutes for this section

Booklet 1 – Attempt Questions 11–25 (40 marks)
Booklet 2 – Attempt Questions 26–37 (50 marks)

Instructions

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on page 21. If you use this space, clearly indicate which question you are answering.

Q	Marks
11	/2
12	/2
13	/3
14	/3
15	/2
16	/3
17	/2
18	/1
19	/2
20	/2
21	/3
22	/2
23	/5
24	/4
25	/4
Total	**/40**

Please turn over

Question 11 (2 marks)

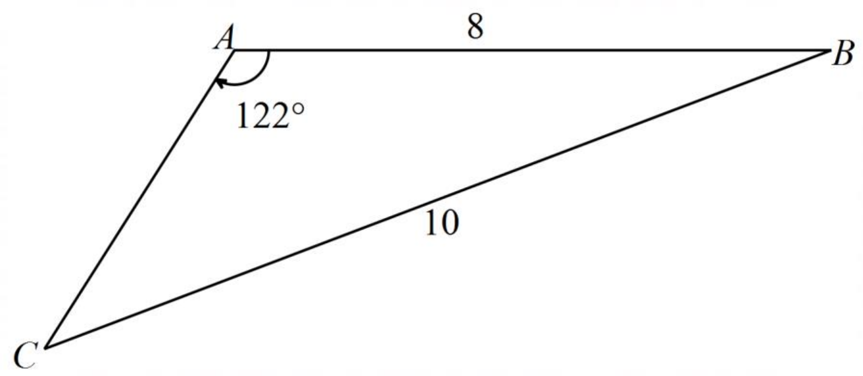
Solve for x **2**

$$\frac{2x+5}{6} - x = \frac{1}{3}$$

Question 12 (2 marks)

In $\Delta\, ABC,$ AB=8, BC=10 and $\angle CAB = 122°$. **2**

Find angle ACB, answer to the nearest degree.



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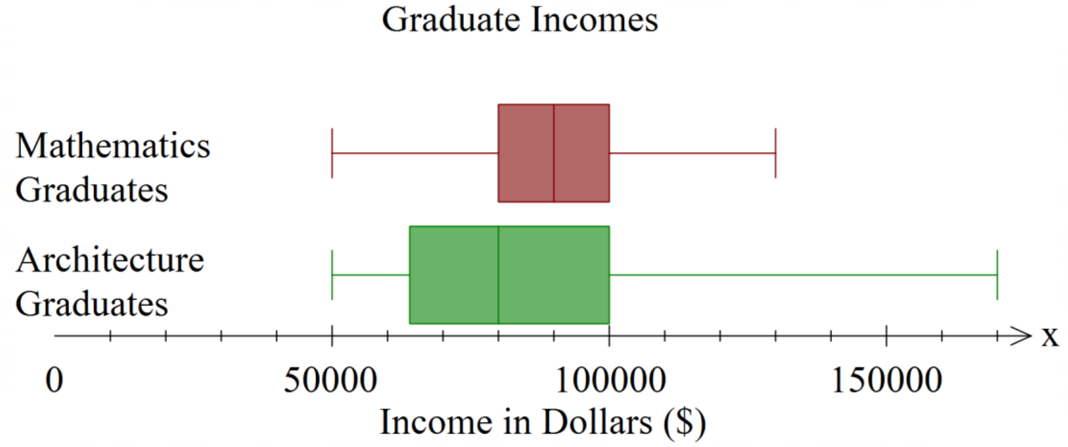
Question 13 (3 marks)

The income of recent graduates from either a Mathematics or Architecture degree has been sketched on the box plot below.



(a) An architecture student finds a graduate position with a starting salary of $175000 per year. Determine if this salary is an outlier. **2**

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(b) A researcher claimed that the same number of graduates earned between 80000 and 100000 for both Architecture and Mathematics degrees. What would need to be true about the data sets in order for the researcher to be correct? **1**

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Question 14 (3 marks)

The owners of Wilson Tennis Racquets use the equation $pq = T$ to ensure that the price of their racquets is appropriate to the current market.

Where p is the price of a racquet, q is the quantity sold per year and T is a constant.

(a) The price of a racquet last year was $p = \$100$ and the quantity sold was $q = 4000$. Find T. **1**

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(b) Sketch the graph of the demand curve with q on the horizontal axis and p on the vertical axis. **2**

Question 15 (2 marks)

Use the trapezoidal rule with three subintervals to find an approximation for the following definite integral, correct to two significant figures. **2**

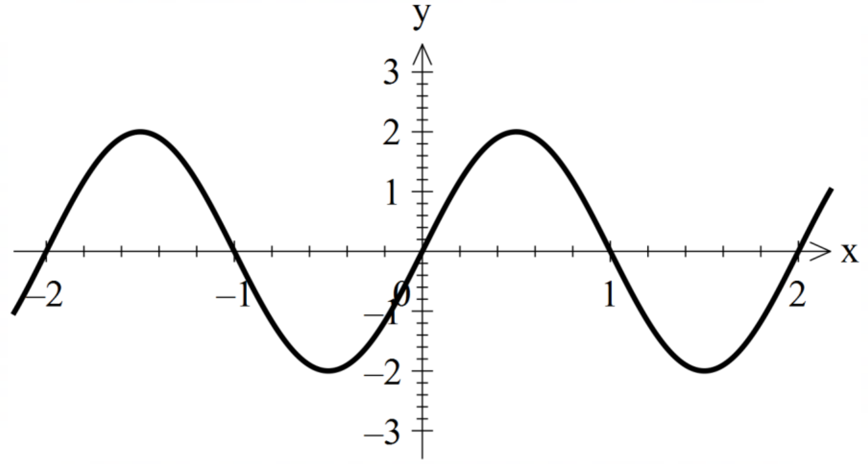
$$\int_0^6 \ln(x+1)\,dx$$

Question 16 (3 marks)

Find the equation of the tangent to the curve $y = 3x\log_e x$ at $(e, 3e)$. **3**

Question 17 (2 marks)

The following curve $g(x)$ is a transformation of the curve $f(x) = \sin x$. **2**
Let $g(x) = k sin(ax)$. Find the values for k and a, given $a > 0$.



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Question 18 (1 marks)

The first term of an AP is 17 and the 90^{th} term is 2064. Find the common difference d. **1**

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Question 19 (2 marks)

Evaluate the following definite integral. **2**

$$\int_1^2 \frac{x^2+1}{x^3}\,dx$$

Question 20 (2 marks)

If $f(x+1) = x^2 + 3x + 5$, find $f(x)$. **2**

Question 21 (3 marks)

Solve $2\cos^2(x) + \sqrt{3}\cos(x) = 0$ for $0 \leq x \leq 2\pi$. **3**

Question 22 (2 marks)

Differentiate $y = 3(1 + e^{2x})^3$. **2**

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Question 23 (5 marks)

The displacement of a particle is given by the equation

$$x = 11 + 8t - 10\sqrt{t^2 + 4}.$$

(a) Find the point at which the particle is at rest. **3**

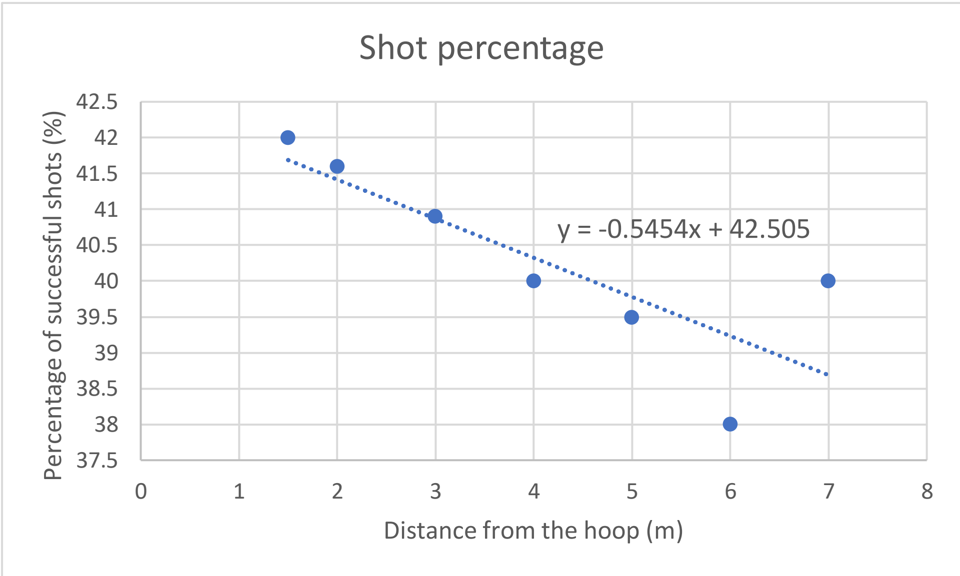
(b) Find the distance travelled in the first 10 seconds. **2**
(Give your answer to the nearest whole number.)

Question 24 (4 marks)

A basketball player keeps track of the percentage of shots that she successfully makes, depending on how many metres she is away from the basket.

Distance (m)	1.5	2	3	4	5	6	7
Percentage (%)	42	41.6	40.9	40	39.49	38	40

She plots the points on the scatter plot below and calculated the least squares regression line, $y = -0.5454x + 42.505$, where x is the distance in metres from the basketball hoop and y is the percentage of successful shots.



(a) By calculating Pearsons Correlation coefficient describe the nature of the correlation. **2**

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(b) The basketball player wants to calculate the percentage of shots she would score from the halfway line which is 14.3m from the hoop.
Use the least squares regression line to calculate the percentage of the success shot to the whole number. **1**

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(c) Explain whether it is reasonable to use this least squares regression line to determine the answer in part b? **1**

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Question 25 (4 marks)

A geometric series has an infinite sum of 56870. The common ratio of this series is $\frac{9}{11}$.

(a) Show that the first term of the series is 10340. **1**

(b) Find the smallest number of terms in the series that need to be added so that the sum exceeds 55 000. **3**

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Section II extra writing space

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Section II extra writing space

If you use this space, clearly indicate which question you are answering.

MATHEMATICS DEPARTMENT

Student Number

2023	Year 12 Trial Examination

Mathematics Advanced

Section II Answer Booklet 2

07/08/2023

Booklet 2 – Attempt Questions 26–37 (50 marks)

Q	Marks
26	/2
27	/3
28	/4
29	/5
30	/3
31	/8
32	/5
33	/3
34	/2
35	/3
36	/6
37	/6
Total	**/50**

Instructions

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on page 39. If you use this space, clearly indicate which question you are answering.

Please turn over

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Question 26 (2 marks)

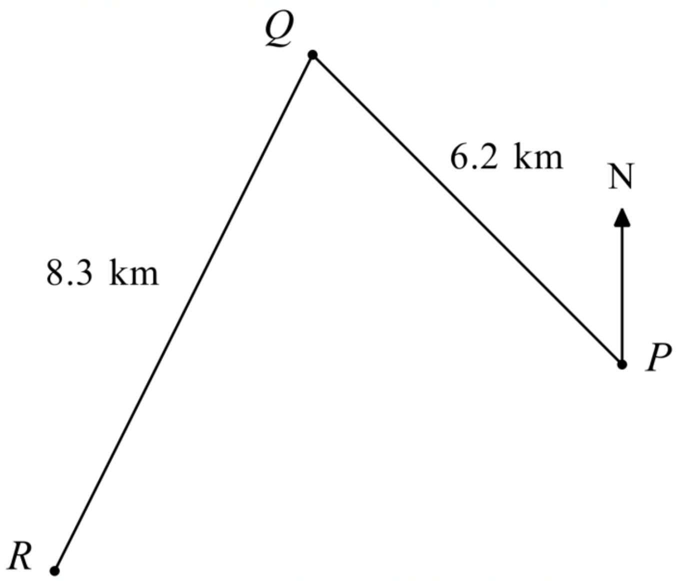
f is the function such that $f(x) = 2\ln(x)$ **2**

g is the function such that $g(x) = e^x$

Solve $g\big(f(x)\big) = 16$

Question 27 (3 marks)

A hiker is planning a journey over three days. The hiker starts at the point P and travels 6.2 km directly north-west to point Q. The hiker then travels 8.3 km on a bearing of 244°T to point R. **3**



Determine the shortest distance the hiker needs to travel to return from point R to point P, leaving your answers in km, correct to 1 decimal place.

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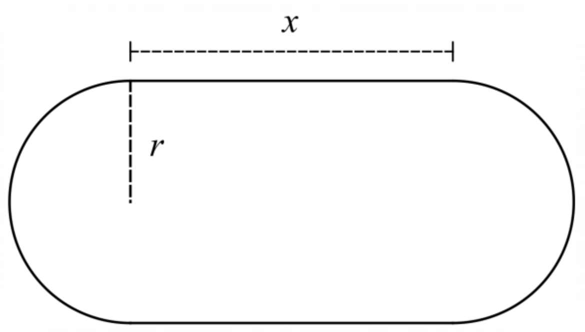
Question 28 (4 marks)

An area of land is being made using 100 m of fencing. It is being designed as a rectangle of length x metres, with two semicircles at each end with a radius of r metres. **4**



Find the maximum area of land that can be made, showing why this area is a maximum.

Question 29 (5 marks)

A café purchases an automatic coffee machine that dispenses a cup of coffee in any style without the need for a barista. The volume of the cups of coffee the machine produces are normally distributed with a mean volume of 240mL and a standard deviation of 5mL.

(a) If the café makes 150 Cappuccinos a day and has bought cups that are advertised as 250mL. How many cups would you expect to overflow in a day? **2**

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(b) After purchasing the coffee machine, the Café finds that, on average, when producing 150 cups of coffee a day 6.75 cups will overflow in a 30 day period. Based on this information estimate the actual volume of the cups they have purchased? **3**

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Question 30 (3 marks)

The hyperbola function $y = \frac{a}{b - x} - 1$ passes the coordinate point of (2,1) and $y'(1) = \frac{1}{2}$. Find a and b. **3**

Question 31 (8 marks)

The function $y = 2x^5 + 5x^2 - 8$

(a) Find all stationary points and determine their nature. **3**

(b) Find any points of inflection **2**

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Do NOT write in this area.

(c) Hence, sketch the curve. You **do not** need to show the x-intercepts. **3**

Question 32 (5 marks)

The mass, m grams, of a leaf t days after it has been picked from a tree is given by

$m = a \times 3^{-kt}$.

Where a and k is are positive constants.

When the leaf is picked from the tree, its mass is 7.5 grams and 4 days later its mass is 2.5 grams.

(a) Show that $k = 0.25$. **2**

(b) Find the value of t when $\frac{dm}{dt} = -0.6\, ln\, 3$. Give your answer correct to 1 decimal place. **3**

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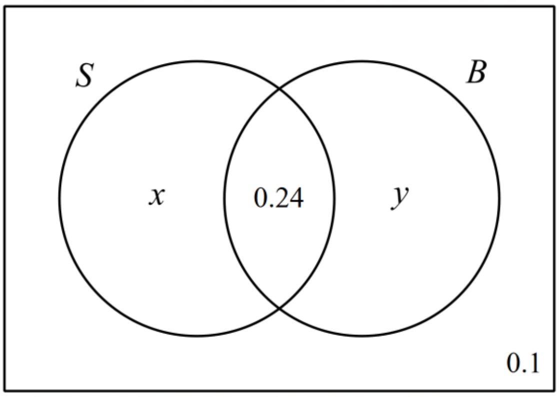
Question 33 (3 marks)

The subjects that a group of university students take is recorded. Out of the group, 24% of the students do both Statistics (S) and Business (B), while 10% do neither of them.



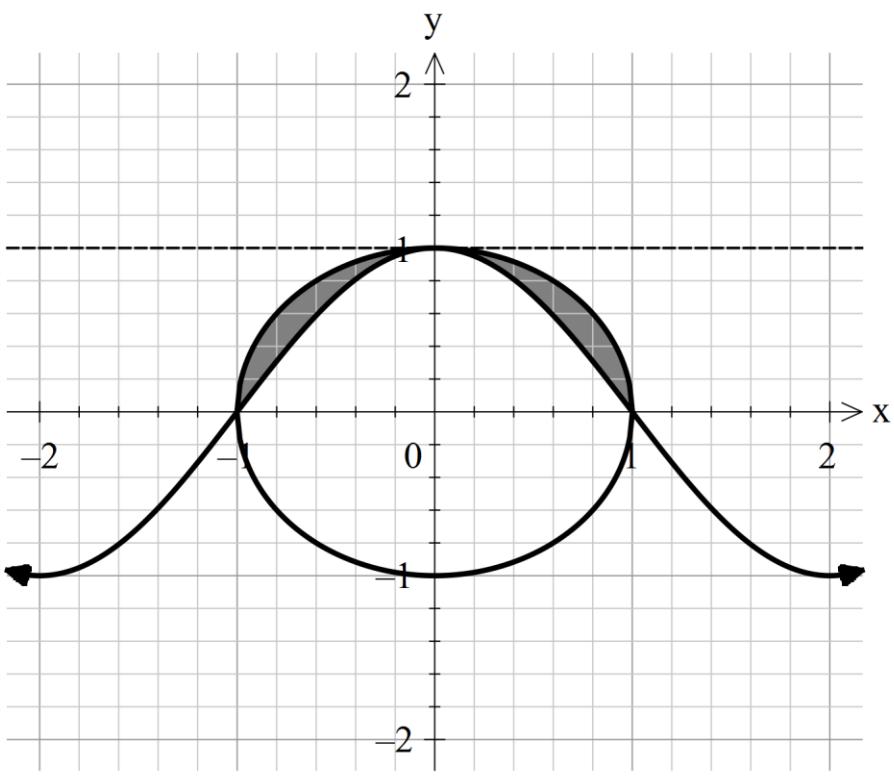
(a) It is known that the number of students in the group that take Statistics is twice the number of students that take Business. **2**

Find the values of x and y in the Venn diagram above.

(b) If a student is selected at random, find the probability that they are not taking Statistics given that they are studying Business? **1**

Question 34 (3 marks)

The curve $x^2 + y^2 = 1$ and $y = \cos(\frac{\pi}{2}x)$ are sketched below. **3**



Find the exact area of the shaded region.

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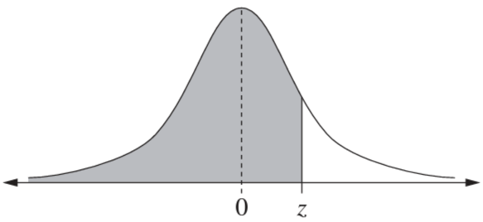
Question 35 (2 marks)

A large group of students sat a Mathematics test. The results of this test were normally distributed with mean score of 66. It is known that the upper quartile of the results was a score of 80. **2**

Calculate the standard deviation of the data set, leaving your answer to the nearest whole number.

Use the table of z-scores below to answer this question.

Table of values $P(Z \le z)$ for the normal distribution $N(0, 1)$



Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621

Question 36 (6 marks)

The hours of daylight in Sydney can be modelled by

$$y(t) = 2.24\sin\left(\frac{\pi t}{6} + 1.34\right) + 12.19$$

Where y is the number of hours of daylight and t is the months of the year, each month being represented with a whole number, e.g. t =1 represents January 1st.

(a) In this model what is the maximum number of daylight hours in one day. **1**

(b) Find the month that has the longest day in terms of number hours of daylight. **2**

Do NOT write in this area.

(c) A scientist needs 13 hours of continuous daylight for an experiment. Between which dates should she be looking to run her experiment? **3**

Question 37 (6 marks)

Consider the function $f(x) = (x - 38)e^{k-x}$, where k is a constant.

(a) By differentiating $f(x)$, find $\int (x - 39)e^{k-x}dx$ **2**

(b) A chocolate bar is being manufactured by a machine which produces bars weighing between 39 g and 44 g. The weight of a bar produced can be modelled using the continuous random variable X which has a probability density function: **2**

$$p(x) = \begin{cases} (x-39)e^{k-x}, & \text{for } 39 \le x \le 44 \\ 0, & \text{for all other } x \text{ values} \end{cases}$$

Show that $k = 44 - \ln(e^5 - 6)$.

(c) It is known that $P(39 \leq X \leq 40) < 0.3$. Show that the median weight of the chocolate bar produced is between 40 and 41 grams. **2**

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End of paper

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

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Do NOT write in this area.

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

MATHEMATICS DEPARTMENT

Student Number

2023 **Year 12 Trial Examination**

Mathematics Advanced

07/08/2023

General Instructions	• Reading time – 10 minutes • Working time – 3 hours • Write using blue or black pen • Calculators approved by NESA may be used • A reference sheet is provided • For questions in Section II, show relevant mathematical reasoning and/or calculations • No white-out may be used
Total Marks: 100	**Section I - 10 marks** (pages 3–10) • Allow about 15 minutes for this section **Section II - 90 marks** (pages 11–44) • Allow about 2 hours and 45 minutes for this section

This question paper must not be removed from the examination room.

This assessment task constitutes 40% of the course.

Section I

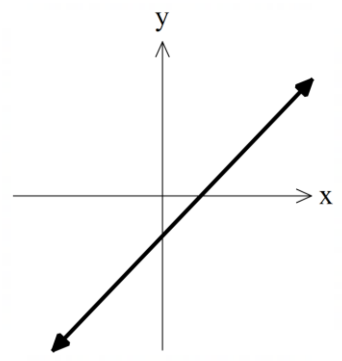
10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section.

Use the multiple-choice sheet for Question 1–10.

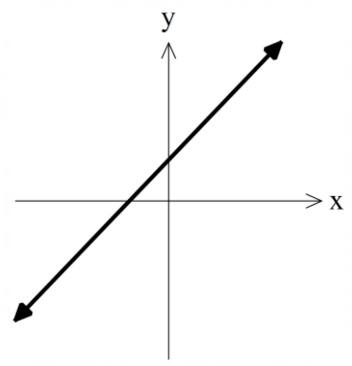
---

1 Which one of the following could be $y = -x + 3$?

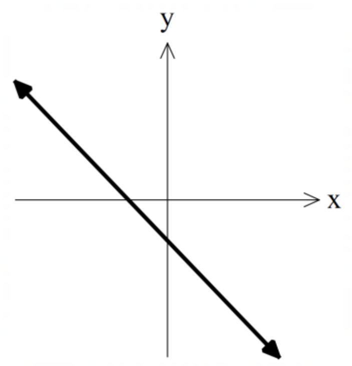
A.



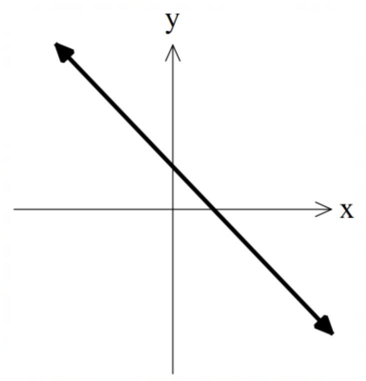
B.



C.



D.



2 The probability distribution of the random variable X is given in the table below. Calculate $E(X)$

x	2	3	4	5
$P(X = x)$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$	

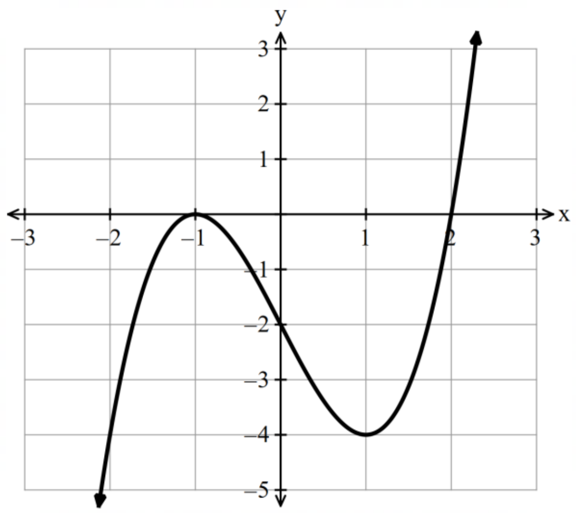
A. $\frac{1}{6}$

B. 2

C. $\frac{8}{3}$

D. $\frac{7}{2}$

3 Which of the following equations best describes the graph below?



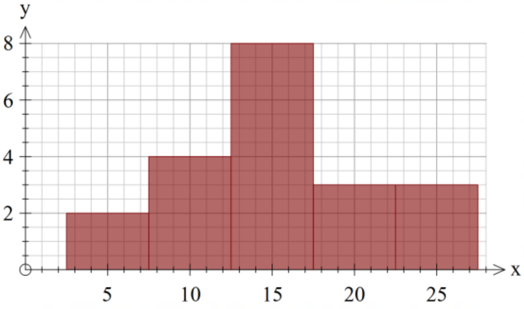
A. $y = (x+1)^2(x-2)$

B. $y = (x+1)(x-2)^2$

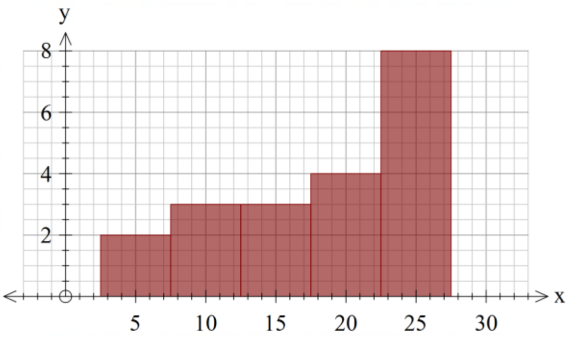
C. $y = (x-1)^2(x+2)$

D. $y = (x-1)(x+2)^2$

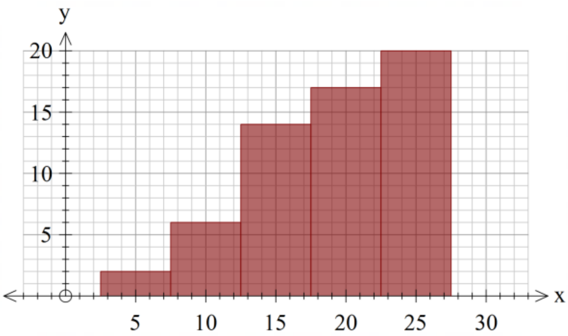
4 A frequency histogram is given below where the y-axis represents the frequency. Which one of the following is the corresponding cumulative frequency histogram?



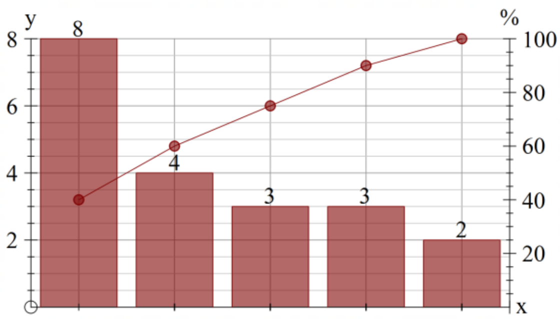
A.



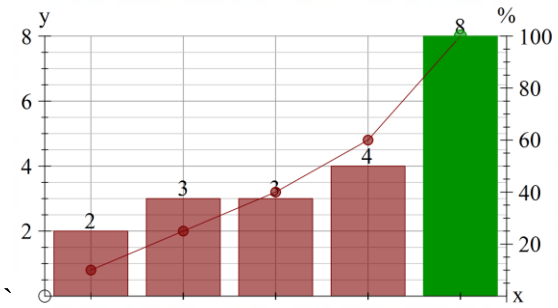
B.



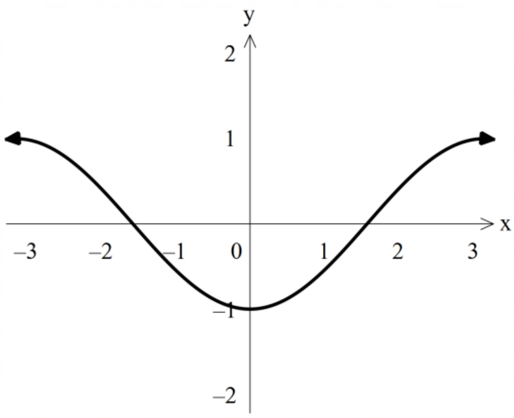
C.



D.

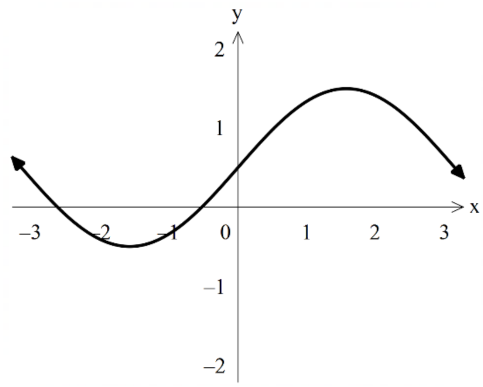


5 The gradient function, $f'(x)$ is sketched below.

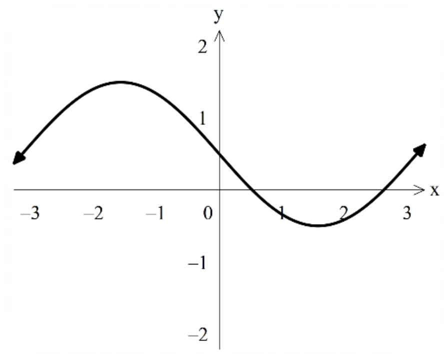


Which one is the possible graph of $f(x)$?

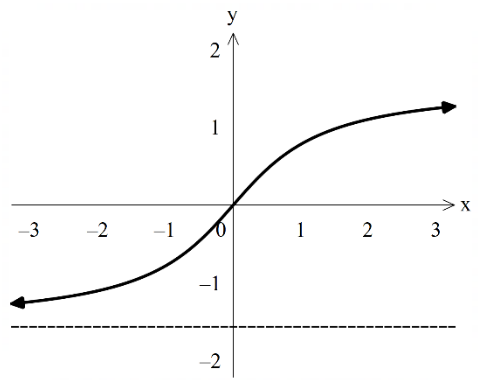
A.



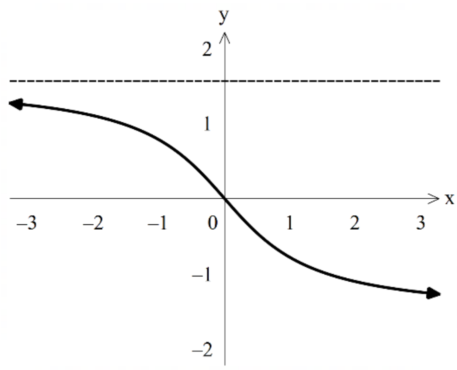
B.



C.



D.



6 A function is defined as:

$$f(x) = x^2 - 6x + 10 \text{ for } x \in [-1, 3).$$

What is its range?

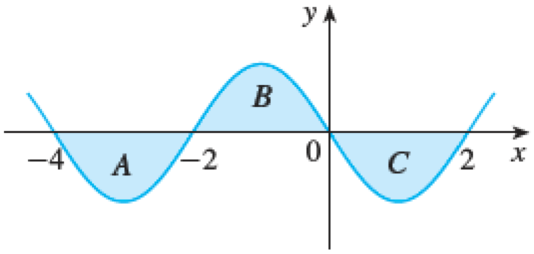
A. $y \in (1, \infty)$

B. $y \in [1, \infty)$

C. $y \in (1, 17]$

D. $y \in [1, 17]$

7 Each of the regions A, B and C bound by the graph of f and the x-axis has the same area. If $\int_0^2 [f(x) + 2x + 5]dx = 11$, what is the value of $\int_{-4}^2 f(x)dx$



A. $\int_{-4}^{2} f(x)dx = -6$

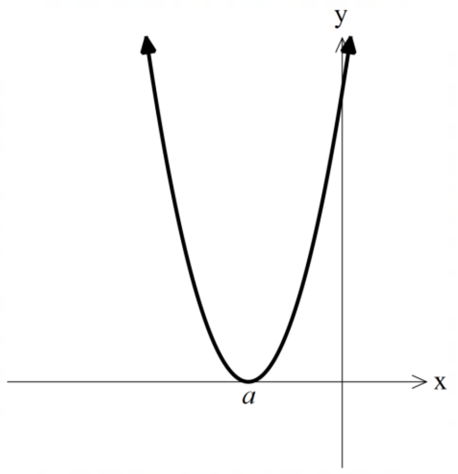
B. $\int_{-4}^{2} f(x)dx = 6$

C. $\int_{-4}^{2} f(x)dx = -3$

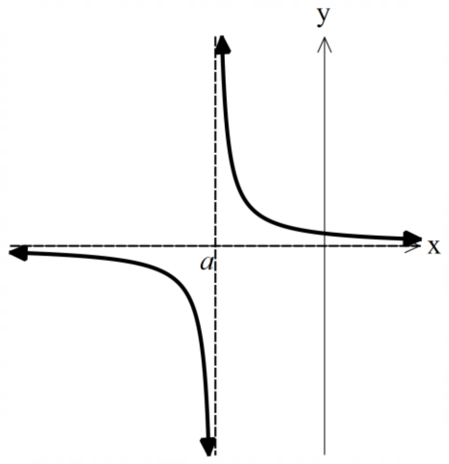
D. $\int_{-4}^{2} f(x)dx = 3$

8. The graphs of $f(x)$ and $g(x)$ are shown below. It is known that the turning point of $f(x)$ occurs where $x = a$ and the vertical asymptote of $g(x)$ is $x = a$.

$\boldsymbol{y = f(x)}$

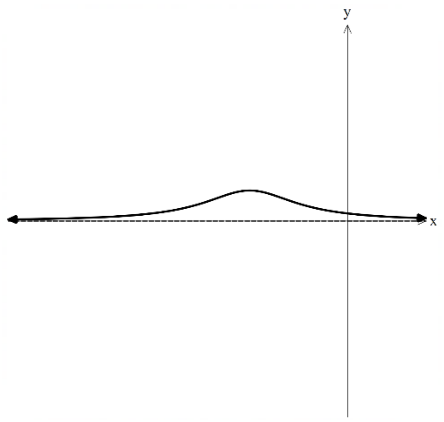


$\boldsymbol{y = g(x)}$

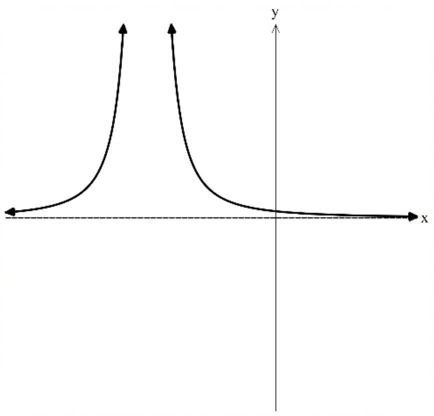


Which one of the following graphs best represents the graph of $g(f(x))$?

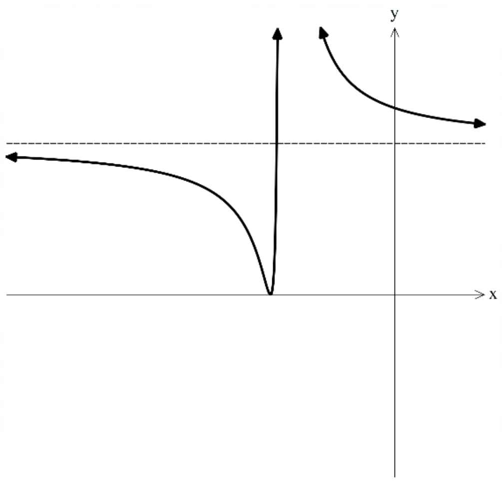
A.



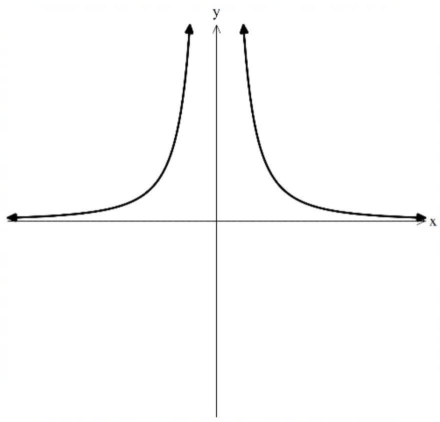
B.



C.



D.



9 What is $\int \frac{1}{1-x} dx$?

A. $\ln|1-x| + c$

B. $\ln\left|\frac{1}{1-x}\right| + c$

C. $\ln 1 - \ln|x| + c$

D. $-\ln|x| + c$

10 A probability density function is given below:

$$f(x) = \begin{cases} x-1, & 1 \le x \le a \\ 0, & else\ where \end{cases}$$

Which one of the following is the correct way to find the value of a

A. The larger root of $a^2 - 2a - 1 = 0$

B. The smaller root of $a^2 - 2a - 1 = 0$

C. The larger root of $a^2 + 2a - 1 = 0$

D. The smaller root of $a^2 + 2a - 1 = 0$

1.	D
2.	D
3.	A
4.	B
5.	B
6.	C
7.	C
8.	A
9.	B
10.	A
	Marker's Feedback: Most students got first 5 questions correct. Only a few students got Q8 correct.

End of Section I

KILLARA HIGH SCHOOL
MATHEMATICS DEPARTMENT

Student Number

2023 **Year 12 Trial Examination**

Mathematics Advanced

Section II Answer Booklet 1

07/08/2023

Q	Marks
11	/2
12	/2
13	/3
14	/3
15	/2
16	/3
17	/2
18	/1
19	/2
20	/2
21	/3
22	/2
23	/5
24	/4
25	/4
Total	**/40**

Section II

90 Marks
Attempt Questions 11–37
Allow about 2 hours and 45 minutes for this section

Booklet 1 – Attempt Questions 11–25 (40 marks)
Booklet 2 – Attempt Questions 26–37 (50 marks)

Instructions

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on page 21. If you use this space, clearly indicate which question you are answering.

Please turn over

Do NOT write in this area.

Do NOT write in this area.

Question 11 (2 marks)

Solve for x **2**

$$\frac{2x+5}{6} - x = \frac{1}{3}$$

$$\begin{aligned} 2x + 5 - 6x &= 2 \\ 3 &= 4x \\ x &= \frac{3}{4} \end{aligned}$$

1 mark – made 1 mistake, arithmetic, or by not multiplying the x on the LHS along with anything else.
2 mark – correct answer from correct working

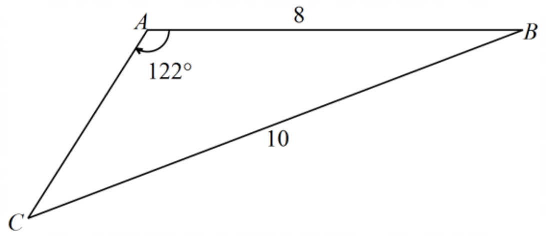
Marker's Feedback:

Overall well done.

Question 12 (2 marks)

In $\Delta\, ABC$, AB=8, BC=10 and $\angle CAB = 122°$. **2**

Find angle ACB, answer to the nearest degree.



$$\begin{aligned} \frac{\sin(C)}{8} &= \frac{\sin(122°)}{10} \\ \sin(C) &= \frac{8\sin(122°)}{10} \\ C &= \sin^{-1}(\,8\sin(122°) \div 10\,) \\ &= 42.72\,... \\ &= 43° \end{aligned}$$

1 mark – sets up the the sign rule.

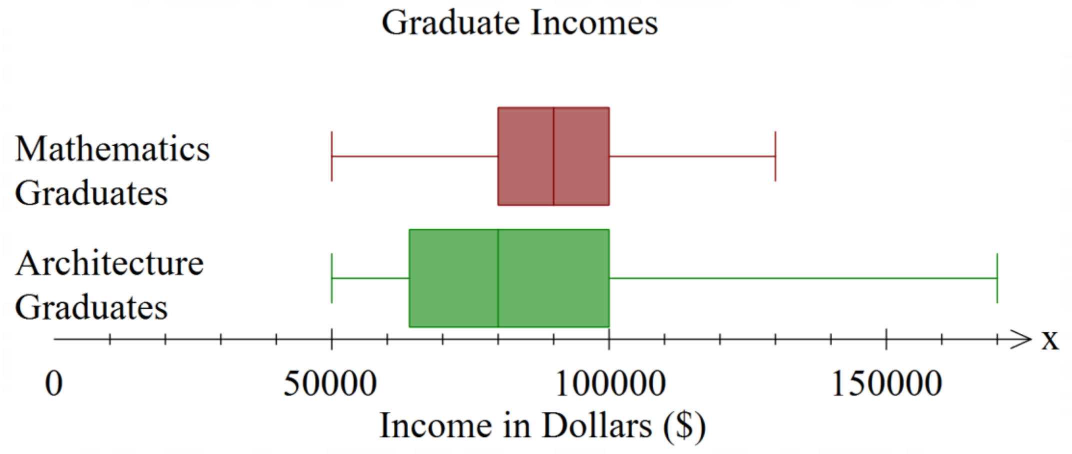
2 marks – correct answer from correct working

Marker's Feedback:

Most students got one mark with correct setting out sine rule. A number of students attempted to use cosine rule.

Question 13 (3 marks)

The income of recent graduates from either a Mathematics or Architecture degree has been sketched on the box plot below.



(a) An architecture student finds a graduate position with a starting salary of $175000 per year. Determine if this salary is an outlier. **2**

$IQR = 100000 - 65000$
$= 35000$
$Upper\ fence\ for\ an\ outlier$
$= Q_3 + IQR$
$= 100000 + 35000 \times 1.5$
$= 152500$
$\therefore \$175000\ would\ be\ an\ outlier\ as\ it\ is$

$above\ the\ upper\ fence\ for\ an\ outlier.$

1 mark – calculates the interquartile range correctly

2 marks – calculates the "upper fence" (or equivalent) AND makes a comparison to $175000

(b) A researcher claimed that the same number of graduates earned between 80000 and 100000 for both Architecture and Mathematics degrees. What would need to be true about the data sets in order for the researcher to be correct? **1**

There would need to be exactly twice as many Architecture graduates as

Marker's Feedback:

(a) Quite a few students did not know how to determine an outlier.
(b) Less half of students able to identify that number of architecture are needed to be doubled, in order to have the same number of graduates earned between 80000 and 100000.

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Question 14 (3 marks)

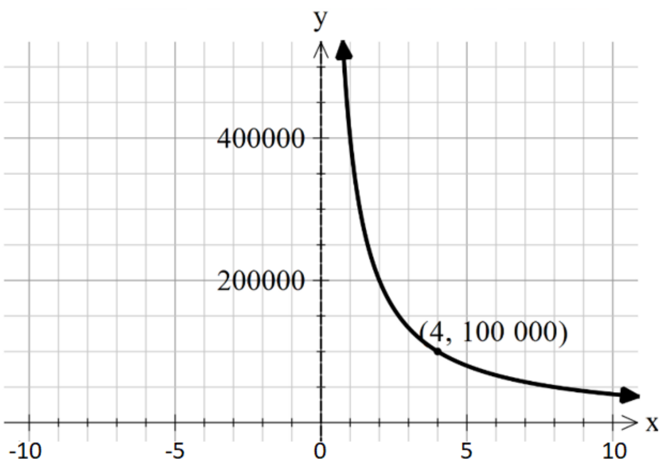
The owners of Wilson Tennis Racquets use the equation $pq = T$ to ensure that the price of their racquets is appropriate to the current market.

Where p is the price of a racquet, q is the quantity sold per year and T is a constant.

(a) The price of a racquet last year was $p = \$100$ and the quantity sold was $q = 4000$. Find T. **1**

$$T = 100 * 4000$$
$$= 400000$$

(b) Sketch the graph of the demand curve with q on the horizontal axis and p on the vertical axis. **2**



1 mark – Shape and mostly even scale
2 marks – Shape, even scale and one point labelled.

Marker's Feedback:
(a) Done very well.
(b) Quite a few students drew a straight line instead of a hyperbola curve.

Question 15 (2 marks)

Use the trapezoidal rule with three subintervals to find an approximation for the following definite integral, correct to two significant figures. **2**

$$\int_0^6 \ln(x+1)\, dx$$

$h = 2\ (6 \div 3\ (sub - intervals))$

$$A \approx \frac{2}{2}[\ln 1 + \ln 7 + 2(\ln 3 + \ln 5)]$$

(**One mark** for correctly substituting)

7.362010551 ...

7.4 (2 Significant Figures) (no units required)

(**One mark** for answer written for 2 SF)

most students achieved one mark for ecf or the value for the height. A review of the formula is advised and how to break into the sub intervals.

Question 16 (3 marks)

Find the equation of the tangent to the curve $y = 3x \log_e x$ at $(e, 3e)$. **3**

$$\frac{dy}{dx} = vu' + uv'$$

$$\frac{dy}{dx} = 3\log_e x + 3$$

$= 3(\log_e x + 1)$ **(One mark)**

$$At\ x = e, \frac{dy}{dx} = 3(\log_e e + 1)$$

$\frac{dy}{dx} = 6$ **(One mark)**

$$at\ x = e, y = 3e$$

$$y - 3e = 6(x - e)$$

$y = 6x - 3e$ **(One mark)**

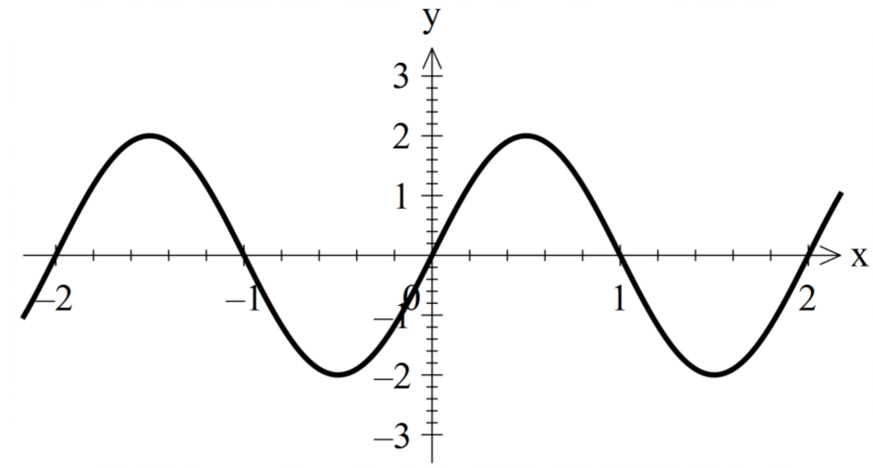
The main issue as substituting the value $x = e$. At least two marks were achieved by most. $\log_e e = 1$ many forgot this and a mark was taken off.

Do NOT write in this area.

Do NOT write in this area.

Question 17 (2 marks)

The following curve $g(x)$ is a transformation of the curve $f(x) = \sin x$. **2**
Let $g(x) = ksin(ax)$. Find the values for k and a, given $a > 0$.



$k = 2$ **(One mark)** most got this mark.

$$Period = \frac{2\pi}{a}$$

$$2 = \frac{2\pi}{a}$$

$a = \pi$ **(One mark)** many did not fix up their algebra.

$$g(x) = 2\sin(\pi x)$$

Question 18 (1 marks)

The first term of an AP is 17 and the 90th term is 2064. Find the common difference d. **1**

$$T_{90} = a + (n - 1)d$$

$$17 + (90 - 1)d = 2064$$

$d = 23$ **(One mark)**

Well done overall.

Question 19 (2 marks)

Evaluate the following definite integral. **2**

$$\int_1^2 \frac{x^2+1}{x^3}\,dx$$

$$= \int_1^2 \frac{1}{x} + \int_1^2 \frac{1}{x^3}\,dx$$

$$= [\ln x]_1^2 - \left[\frac{-1}{2x^2}\right]_1^2 \textbf{ (One mark)}$$

$$= (\ln 2 - \ln 1) + \left(-\frac{1}{8} + \frac{1}{2}\right)$$

$$= \ln 2 + \frac{3}{8} \textbf{ (One mark)}$$

many did not break up the fraction.

most got one of the two parts.

some left as a decimal

Question 20 (2 marks)

If $f(x+1) = x^2 + 3x + 5$, find $f(x)$. **2**

Let $k = x + 1$

$x = k - 1$ **(One mark)**

$f(x) = f(k-1) = (k-1)^2 + 3(k-1) + 5$

$f(x) = x^2 + x + 3$ **(One mark)**

many did not know where to start.

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Do NOT write in this area.

Question 21 (3 marks)

Solve $2\cos^2(x) + \sqrt{3}\cos(x) = 0$ for $0 \le x \le 2\pi$. **3**

$\cos x(2\cos x + \sqrt{3}) = 0$ **(One mark)** many did not split the function.

$\cos x = 0 | \cos x = -\frac{\sqrt{3}}{2}$

(One mark)$x = \frac{\pi}{2}, \frac{3\pi}{2} | x = \frac{5\pi}{6}, \frac{7\pi}{6}$ **(One mark)**

does it work for the original domain!

where is the angle +ve or -ve.

Question 22 (2 marks)

Differentiate $y = 3(1 + e^{2x})^3$. **2**

$let\, u = 1 + e^{2x}$

$u' = 2e^{2x}$**(One mark)**

$y = 3(u)^3$

$y' = 9u^2$

$y' = 9(1 + e^{2x}) \times 2e^{2x}$

well done

$y' = 18e^{2x}(1 + e^{2x})$ **(One mark)**

Question 23 (5 marks)

The displacement of a particle is given by the equation.

$$x = 11 + 8t - 10\sqrt{t^2 + 4}.$$

(a) Find the point at which the particle is at rest. **3**

$\dot{x} = 8 - 5(u)^{-0.5}$ let $u = t^2 + 4$

$u' = 2t$ **(One mark)**

$\dot{x} = 8 - \frac{10t}{\sqrt{t^2+4}} = 0$

$t^2 = \frac{64}{9}$

$t = \pm\frac{8}{3}$ **(One mark)**

$at\ rest\ t = 2\frac{2}{3}$ seconds **(One mark)**

Algebra caused most of the issues

(b) Find the distance travelled in the first 10 seconds. **2**
(Give your answer to the nearest whole number.)

From (a) the particle comes to rest at $t = 2\frac{2}{3}$ seconds. So distance travelled in the first 10 seconds will be based on three factors. At $t = 0$, $t = 2\frac{2}{3}$ and $t = 10$ seconds. There is a change in direction at (-9). **(One mark)** is allocated for students who have identified this. The distance then should be calculated based on:

$t = 0, \quad x = -9$

$t = 2\frac{2}{3}, \quad x = -1$

$t = 10, \quad x = 91 - 20\sqrt{26}$

motion need to be revised.

Distance is approximately $\approx |-1-(-9)| + |91 - 20\sqrt{26} - (-1)|$ $\approx$18 units
(One mark)

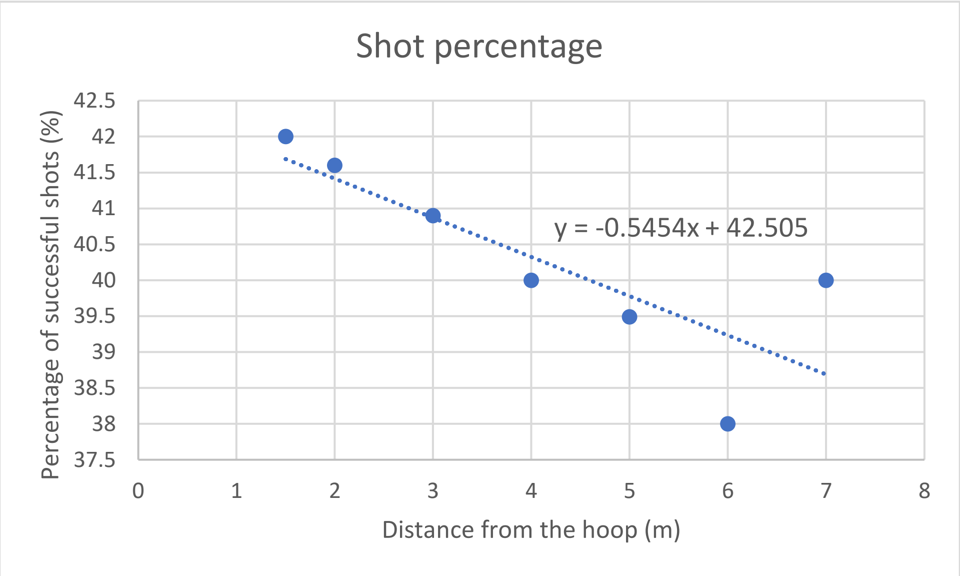
Do NOT write in this area.

Question 24 (4 marks)

A basketball player keeps track of the percentage of shots that she successfully makes, depending on how many metres she is away from the basket.

Distance (m)	1.5	2	3	4	5	6	7
Percentage (%)	42	41.6	40.9	40	39.49	38	40

She plots the points on the scatter plot below and calculated the least squares regression line, $y = -0.5454x + 42.505$, where x is the distance in metres from the basketball hoop and y is the percentage of successful shots.



(a) By calculating Pearsons Correlation coefficient describe the nature of the correlation. **2**

$r = -0.8233$ 1 mark: correct answer for r.

Strong negative linear correlation 1 mark: correct description.

Feedback: Most students could use their calculators to find the correlation coefficient. However, very few made a supported statement about the correlation. Only a strong negative correlation has been accepted.

(b) The basketball player wants to calculate the percentage of shots she would score from the halfway line which is 14.3m from the hoop. **1**
Use the least squares regression line to calculate the percentage of the success shot to the whole number.

When $x = 14.3m$ 1 mark: correct answer with solutions.

$y = -0.5454x + 42.505$

$= -0.5454 \times 14.3 + 42.505$

= 35

Feedback: This question has been done pretty well. As the question asked for a whole number percentage the answer should be 35%. Rounding has not been penalised in the question.

(c) Explain whether it is reasonable to use this least squares regression line to determine the answer in part b? **1**

It is not reasonable to use this least squares regression line, as the data only be given up to 7 m from the hoop and we don't know the trend after 7m.

1 mark: correct reasoning.

Feedback: Most students mentioned extrapolation. Other explanations have not been accepted.

Do NOT write in this area.

Question 25 (4 marks)

A geometric series has an infinite sum of 56870. The common ratio of this series is $\frac{9}{11}$.

(a) Show that the first term of the series is 10340. **1**

$S_n = \frac{a}{1-r}$ — 1 mark: correct working.

$a = (1 - r)\, S_n$

$= \frac{2}{11} \times 56870$

$= 10340$

Feedback: This question has been done well in general. A small number of students used the formula for the arithmetic series.

(b) Find the smallest number of terms in the series that need to be added so that the sum exceeds 55 000. **3**

$S_n = \frac{a(1-r^n)}{1-r}$ — 1 mark: correct substitution

$55\,000 = \frac{10\,340\left(1 - \left(\frac{9}{11}\right)^n\right)}{\frac{2}{11}}$ — using $S_n = \frac{a(1-r^n)}{1-r}$.

$55\,000 \times \frac{2}{11} = 10\,340\left(1 - \left(\frac{9}{11}\right)^n\right)$ — 1 mark: correct expression

$1 - \left(\frac{9}{11}\right)^n = \frac{500}{517}$ — in term of power n.

$\left(\frac{9}{11}\right)^n = \frac{17}{517}$

$n = \log_{\frac{9}{11}}\left(\frac{17}{517}\right)$ — 1 mark: correct answer with

$= \frac{\ln\left(\frac{17}{517}\right)}{\ln\left(\frac{9}{11}\right)}$ — working.

≈ 17.02

∴ The smallest number of terms in the series that need to be added so that the sum exceeds 55 000 is 18 terms.

Feedback: A number of students could calculate 17.02 for n, but stated that 17 terms instead of 18 terms were to be used.

A number of students had no idea of how to do this question.

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

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Section II extra writing space

If you use this space, clearly indicate which question you are answering.

KILLARA HIGH SCHOOL
MATHEMATICS DEPARTMENT

Student Number

2023 **Year 12 Trial Examination**

Mathematics Advanced

Section II Answer Booklet 2

07/08/2023

Booklet 2 – Attempt Questions 26–37 (50 marks)

Q	Marks
26	/2
27	/3
28	/4
29	/5
30	/3
31	/8
32	/5
33	/3
34	/2
35	/3
36	/6
37	/6
Total	**/50**

Instructions

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on page 39. If you use this space, clearly indicate which question you are answering.

Please turn over

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Question 26 (2 marks)

f is the function such that $f(x) = 2\ln(x)$ **2**

g is the function such that $g(x) = e^x$

Solve $g\big(f(x)\big) = 16$

$g(2\ln x) = 16$ 1 mark: correct substitution of f(x) into

$e^{2\ln x} = 16$ g(x).

$x^2 = 16$ However, $x > 0$

$x = 4$ $Reject\ x = -4$ 1 mark: correct answer with working

Feedback: Most students did well by simplifying $g\big(f(x)\big) = 16$ into $x^2 = 16$

$2x = 16$ is a common mistake. Another common mistake was not to reject $x = -4$.

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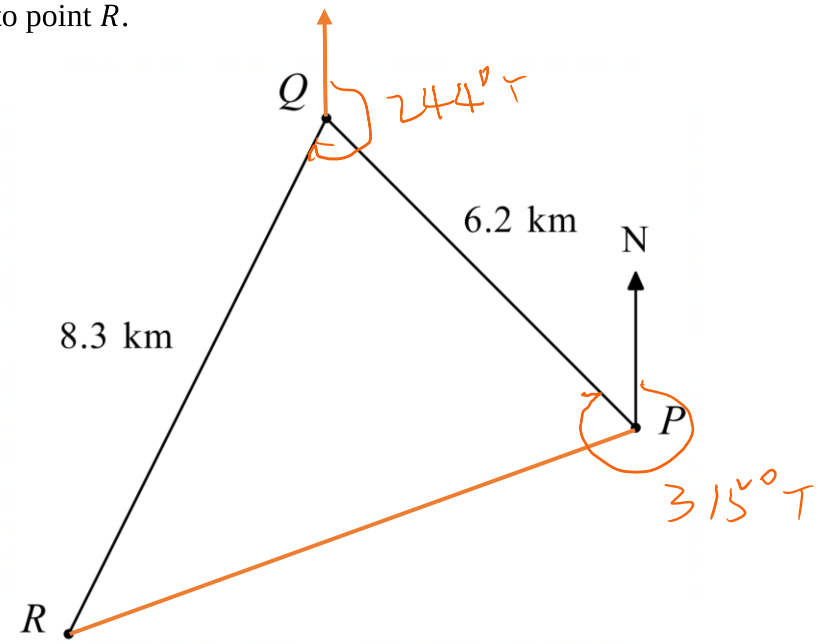
Question 27 (3 marks)

A hiker is planning a journey over three days. The hiker starts at the point P and travels 6.2 km directly north-west to point Q. The hiker then travels 8.3 km on a bearing of 244°T to point R. **3**



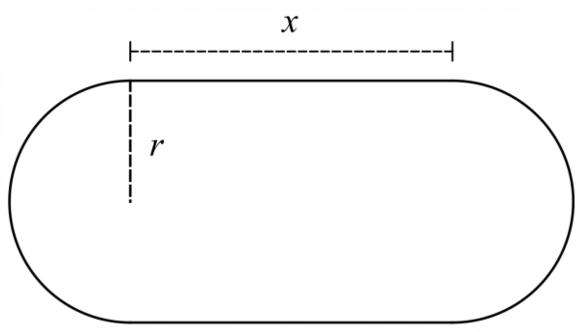
Determine the shortest distance the hiker needs to travel to return from point R to point P, leaving your answers in km, correct to 1 decimal place.

$\angle NPQ = 360° - 315°$	**1 mark:** finding correct angle of
$= 45°$	$\angle PQR$.
$\angle PQN = 180° - 45°$	**1 mark:** correctly applying
$= 135°$	cosine rule.
$\angle PQR = 244° - 135°$	**1 mark:** answer
$= 109°$	
$BP = \sqrt{6.2^2 + 8.3^2 - 2\cos 109° \times 6.2 \times 8.3}$	
$\approx 11.9 km$	

Feedback: A number of students could not find the angle properly from the bearing. Almost all the students knew to apply the cosine rule.

Question 28 (4 marks)

An area of land is being made using 100 m of fencing. It is being designed as a rectangle of length x metres, with two semicircles at each end with a radius of r metres. **4**



Find the maximum area of land that can be made, showing why this area is a maximum.

Method 1:

$100 = 2x + 2\pi r$	**1 mark:** correct express of x in term of r.
$x + \pi r = 50$	**1 mark:** correct total area expression in
$x = 50 - \pi r$	term of r.
	1 mark: correct finding r using 1st
Total Area (A) = $\pi r^2 + 2rx$	derivative and determine r that makes
$= \pi r^2 + 2r(50 - \pi r)$	the area maximum by using either 2nd
$= 100r - \pi r^2$	derivative or 1st derivative table values.
$\frac{d}{dr}A = 100 - 2\pi r$	
$\frac{d}{dr}A = 0,$	**1 mark:** correct maximum area with
$100 - 2\pi r = 0, r = \frac{50}{\pi}$	working.
$\frac{d^2}{dr}A = -2\pi < 0,$	

$\therefore$ The maximum area can be reached when $r = \frac{50}{\pi}$ and $x = 0$, which is a circle, is

$100r - \pi r^2 = 100 \times \frac{50}{\pi} - \frac{2500}{\pi}$

$= \frac{2500}{\pi}\ m^2$

Feedback on next page

Method 2:

$100 = 2x + 2\pi r$	**1 mark:** correct express of r in term of x.
$x + \pi r = 50$	**1 mark:** correct total area expression in
$r = \frac{50-x}{\pi}$	term of x.
	1 mark: correct finding x using 1st
Total Area (A) = $\pi r^2 + 2rx$	derivative and determine x that makes
$= \frac{(50-x)^2}{\pi} + 2x \times \frac{50-x}{\pi}$	the area maximum by using either 2nd
$= \frac{2500-100x+x^2+100x-2x^2}{\pi}$	derivative or 1st derivative table values.
$= \frac{2500-x^2}{\pi}$	**1 mark:** correct maximum area with
$\frac{dA}{dx} = -\frac{2x}{\pi} = 0$	working.
$x = 0$	
$\frac{d^2A}{dx^2} = -\frac{2}{\pi} < 0$	
When $x = 0,\ r = \frac{50}{\pi}$.	
$\therefore$ The maximum area can be reached when $r = \frac{50}{\pi}$ and $x = 0$, which is a circle, is	
$100r - \pi r^2 = 100 \times \frac{50}{\pi} - \frac{2500}{\pi}$	
$= \frac{2500}{\pi}\ m^2$	

Feedback:
This question has been poorly done. Some students could not tell the difference between the perimeter and area, having both in the same algebraic expression. A failure to express x in terms of r or r in terms of x has prevented some students from establishing the area function.

Question 29 (5 marks)

A café purchases an automatic coffee machine that dispenses a cup of coffee in any style without the need for a barista. The volume of the cups of coffee the machine produces are normally distributed with a mean volume of 240mL and a standard deviation of 5mL.

(a) If the café makes 150 Cappuccinos a day and has bought cups that are advertised as 250mL. How many cups would you expect to overflow in a day? **2**

$(X > 250) = P(Z > (250 - 240)/5)$
$= P(Z > 2)$
$= (1 - 0.95)/2$
$= 0.025$

Number of cups expected to overflow a day
$= 0.025 \times 150$
$= 3.75$

1 find the probability
2 use the probability to fund the expected number to overflow.

(b) After purchasing the coffee machine, the Café finds that, on average, when producing 150 cups of coffee a day 6.75 cups will overflow in a 30 day period. Based on this information estimate the actual volume of the cups they have purchased? **3**

$P(X > V) = 6.75/(150 * 30)$
$= 0.0015$
$= P(Z > 3)$

$$z = \frac{x - \bar{x}}{\sigma}$$

$$3 = \frac{x - 250}{5}$$

$$x = 255$$

Therefore the cups are 255mL

`1 mark – find the probability
1 mark – find the related z-score
1 mark – find the related volume

Part a was generally well done, though many students found a z-score and assumed that was the number of cups.
Part b was not well done. Most students did not take into account the 30 when calculating their probability, which subsequently made it difficult to find a z-score. Any z-score other than three had to be well justified in order to be able to used further in the question, as there were no possible calculations to get from 4.5% to a z-score. Very few were able to do this.

Do NOT write in this area.

Question 30 (3 marks)

The hyperbola function $y = \frac{a}{b-x} - 1$ passes the coordinate point of (2,1) and **3**

$y'(1) = \frac{1}{2}$. Find a and b.

Sub $(2, 1)$ into y

$$1 = \frac{a}{b-2} - 1$$
$$2 = \frac{a}{b-2}$$
$$2b - 4 = a \qquad (1)$$

Find y'

$$y = a(b-x) - 1$$
$$y' = a(b-x)^{-2}$$
$$y'(1) = \frac{a}{(b-1)^2} = \frac{1}{2}$$
$$a = \frac{(b-1)^2}{2} \qquad (2)$$

sub (2) into (1)

$$2b - 4 = \frac{(b-1)^2}{2}$$
$$4b - 8 = b^2 - 2b + 1$$
$$0 = b^2 - 6b + 9$$
$$b = 3$$

sub $b = 3$ into (1)

$$a = 2(3) - 4$$
$$a = 2$$

1 mark for correct substitutions to find an equation in terms of a and b
2 marks an attempt to form 1 equation, or a solution without labels e.g. sub 2a=b into (1)
3 marks – a correct, well structed solutions

This question was very poorly done. A startling number of students used the quotient rule and introduced terms to the numerator of y'. Many students also canceled b-1 terms with b-2 terms.
Students also needed to be very careful with their working as, even logically set out working can become difficult to follow as you switch between equations.

Question 31 (8 marks)

The function $y = 2x^5 + 5x^2 - 8$

(a) Find all stationary points and determine their nature. **3**

$$y = 2x^5 + 5x^2 - 8$$
$$y' = 10x^4 + 10x$$
Let $y' = 0$ for stationary points
$$0 = 10x^4 + 10x$$
$$0 = 10x(x^3 + 1)$$
$$x = 0, -1$$
$$y'' = 40x + 10$$
$$y''(0) = 10$$
$\therefore (0, -8)$ is a minimum $(y' = 0, y'' > 0)$
$y''(-1) = -30$
$\therefore (-1, 11)$ *is a maximum* $(y' = 0, y'' < 0)$

1 mark correct first derivative.
2 mark for finding the x-values of the points.
3 marks for testing concavity and giving the points.

Gernerally well done

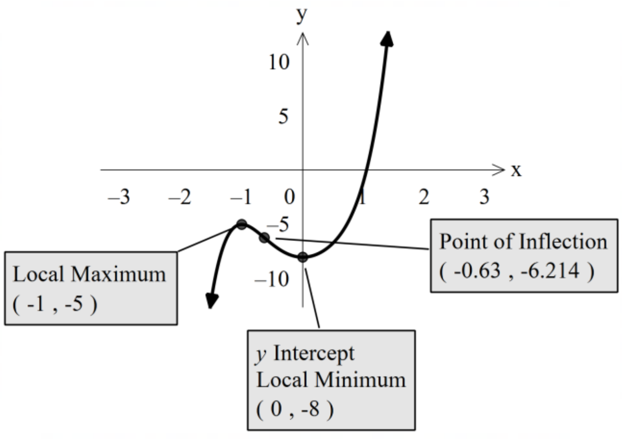
(b) Find any points of inflection **2**

For points of inflection $y'' = 0$
$$40x^3 + 10 = 0$$
$$x = -\sqrt[3]{\frac{1}{4}}$$
Test a point either side
$$y'(0) = 10, \text{from above}$$
$$y'(-1) = -30$$
Therefore there is a change in concavity and
$$\left(-\sqrt[3]{\frac{1}{4}}, -6.214\ldots\right)$$

1 mark for finding the point of inflection
1 mark for finding the point and testing for concavity change

Many students did not test for concavity, or use the values from (a) to justify as a point of inflection.

(c) Hence, sketch the curve. You **do not** need to show the x-intercepts. **3**



1 mark for shape
2 marks for scale and labelled points

Many students did not draw their graph to match their values in part (a) and (b). Even if they drew the correct graph, if it did not match their solutions to (a) and (b) they were awarded no marks.

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Question 32 (5 marks)

The mass, m grams, of a leaf t days after it has been picked from a tree is given by

$m = a \times 3^{-kt}$.

Where a and k is are positive constants.

When the leaf is picked from the tree, its mass is 7.5 grams and 4 days later its mass is 2.5 grams.

(a) Show that $k = 0.25$. **2**

$$\begin{aligned} &Sub\ t = 0, m = 7.5 \\ &7.5 = a \\ &sub\ t = 4, m = 2.5 \\ &2.5 = 7.5 \times 3^{-4k} \\ &\frac{1}{3} = 3^{-4k} \\ &-1 = -4k \\ &k = \frac{1}{4} \\ &\quad = 0.25 \end{aligned}$$

1 mark – for either the value of a or k
2 marks – for both values correct from correct working.

Generally well done.

(b) Find the value of t when $\frac{dm}{dt} = -0.6\ln(3)$. Give your answer correct to 1 decimal place. **3**

$$\begin{aligned} m &= 7.5 \times 3^{-0.25t} \\ \frac{dm}{dt} &= 7.5 \times 3^{-0.25t}(-0.25\ln(3)) \\ &= -\frac{15}{8}\ln(3) \times 3^{-0.25t} \end{aligned}$$

let $\frac{dm}{dt} = -0.6\ln(3)$

$$\begin{aligned} -0.6\ln(3) &= -\frac{15}{8}\ln(3) \times 3^{-0.25t} \\ \frac{8}{25} &= 3^{-0.25t} \\ \frac{\ln\frac{8}{25}}{\ln 3} &= -0.25t \\ t &= \frac{-4\ln\left(\frac{8}{125}\right)}{\ln(3)} \\ &= 4.14\ ... \\ &= 4.1 \end{aligned}$$

1 mark – differentiation with ln(3) or differentiation with ln(3)in wrong position (or missing) but substituted without solving equivalent merit.

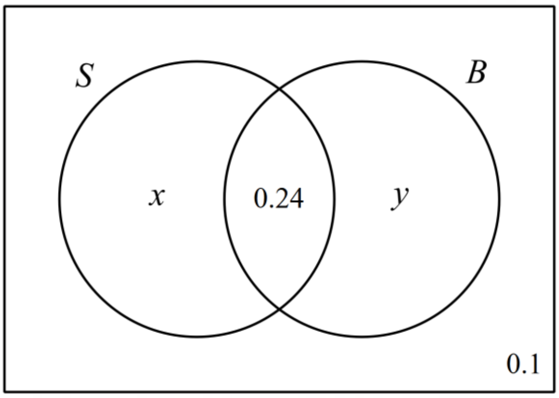
2 marks –
Differentiation with ln(3)in wrong position (or missing), followed by a correct solution given the error.
Correct differentiation, with correct substitution.
Correct answer, but not rounded correctly.
Correct differentiation, correct substitution but either a calculator, arithmetic error or sign error.

3 marks - correct answer from correct working

Generally well done, apart from the initial differentiation, where many students forgot either the 7.5 or the ln(3) or both.

Question 33 (3 marks)

The subjects that a group of university students take is recorded. Out of the group, 24% of the students do both Statistics (S) and Business (B), while 10% do neither of them.



(a) It is known that the number of students in the group that take Statistics is twice the number of students that take Business. **2**

Find the values of x and y in the Venn diagram above.

$S = 2B$

$x + 0.24 = 2y + 0.48 \qquad (1)$

and $x + y + 0.24 + 0.1 = 1$

$x + y = 0.66$

$x = 0.66 - y$

Sub (2) into (1)

$0.66 + 0.24 - y = 2y + 0.48$

$0.42 = 3y$

$0.14 = y$

$y = 0.14$

Sub $y = 0.14$ into (1)

$x + 0.24 = 0.28 + 0.48$

$x = 0.52$

1 mark – setting up the two equations correctly
2 marks correct final answer from correct working.

Too many students assumed $2x = y$

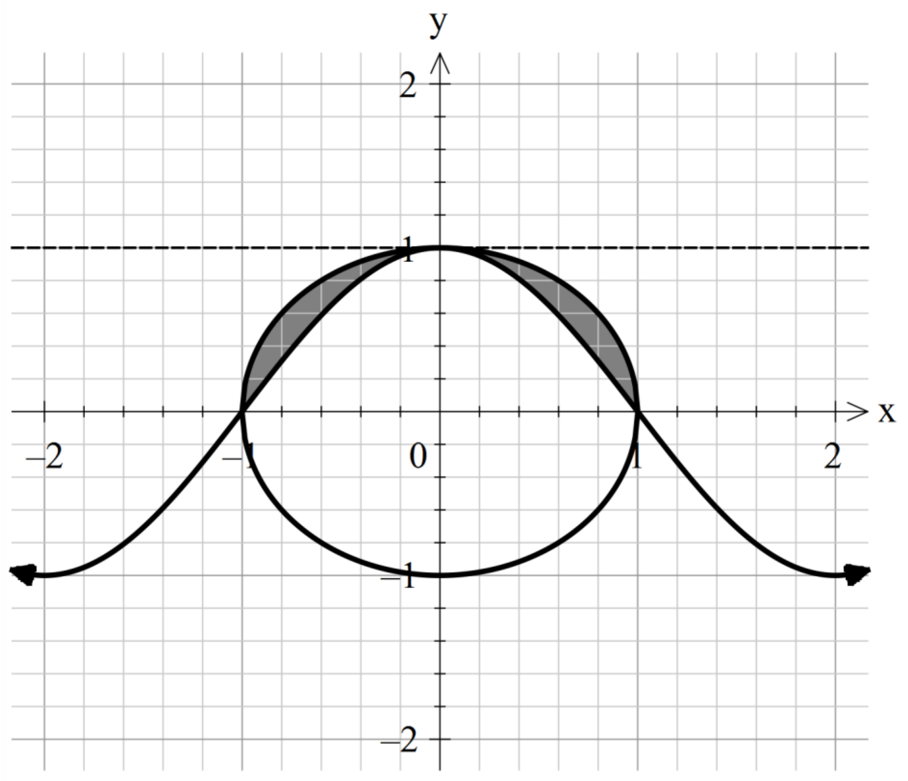
(b) If a student is selected at random, find the probability that they are not taking Statistics given that they are studying Business? **1**

$$P(\bar{S}|B) = \frac{P(\bar{S} \cap B)}{P(B)}$$
$$= \frac{0.1}{0.38}$$
$$= \frac{5}{19}$$

This was generally well done, though many students found P(S|B) rather than $P(\bar{S}|B)$ be very careful you are answering the w

Question 34 (3 marks)

The curve $x^2 + y^2 = 1$ and $y = \cos(\frac{\pi}{2}x)$ are sketched below. **3**



Find the exact area of the shaded region.

$$
\begin{aligned}
A &= \int_{-1}^{1} \sqrt{1-x^2} - \cos\left(\frac{\pi}{2}x\right) dx \\
&= \frac{\pi}{2} - \left[\frac{2}{\pi}\sin\left(\frac{\pi}{2}x\right)\right]_{-1}^{1} \\
&= \frac{\pi}{2} - \frac{2}{\pi}(1 - -(1)) \\
&= \frac{\pi}{2} - \frac{4}{\pi}u^2
\end{aligned}
$$

1 mark- for correct integral set up and and answer with square units

1 mark – for finding the exact area for the semicircle

1 mark – for a correct integral of $\cos\left(\frac{\pi}{2}x\right)$

Feedback:

It was disappointing to see a lot of students getting 0 or 1 for this question. Many students tried to integrate $\sqrt{1-x^2}$ using power laws, which doesn't work in this case. A fair few students also forgot to use the reverse chain rule for $\cos\left(\frac{\pi}{2}x\right)$. There were also many mistakes doing trig exact values, and many students forgot to put square units at the end.

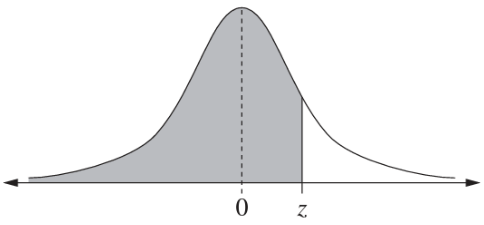
Question 35 (2 marks)

A large group of students sat a Mathematics test. The results of this test were normally distributed with mean score of 66. It is known that the upper quartile of the results was a score of 80. **2**

Calculate the standard deviation of the data set, leaving your answer to the nearest whole number.

Use the table of z-scores below to answer this question.

Table of values $P(Z \le z)$ for the normal distribution $N(0,1)$



Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621

$P(X < 80) = 0.75$

$P(X < 80) = P(Z < 0.67 \text{ or } 0.68)$

$0.67 = \dfrac{80 - 66}{\sigma}$

$\sigma = \dfrac{80 - 66}{0.67}$

$= 20.8955 \ldots (or\ 20.58 \ldots for\ 0.68)$

$= 21$

The standard deviation is 21

1 mark – finding the correct z-score
Or
Using a reasonable incorrect z-score to calculate the standard deviation ($z = 1$ was not paid in this case)
Or
Using the probability from $z = 0.75$

2 marks – correct standard deviation from correct working.

Feedback:

This was not well attempted by the cohort. Many students need to revise and understand what a z-score table is telling you, and be able to explain the different between a z-score of 0.75 vs $p = 0.75$.

Do NOT write in this area.

Question 36 (6 marks)

The hours of daylight in Sydney can be modelled by

$$y(t) = 2.24\sin\left(\frac{\pi t}{6} + 1.34\right) + 12.19$$

Where y is the number of hours of daylight and t is the months of the year, each month being represented with a whole number, e.g. t =1 represents January 1st.

(a) In this model what is the maximum number of daylight hours in one day. **1**

Maximum is $12.19 + 2.24 = 14.43$

(b) Find the month that has the longest day in terms of number hours of daylight. **2**

$$14.43 = 2.24\sin\left(\frac{\pi t}{6} + 1.34\right) + 12.19$$
$$2.24 = 2.24\sin\left(\frac{\pi t}{6} + 1.34\right)$$
$$1 = \sin\left(\frac{\pi t}{6} + 1.34\right)$$
$$\frac{\pi t}{6} + 1.34 = \frac{\pi}{2}, \frac{5\pi}{2}$$
$$\frac{\pi t}{6} = \left(\frac{\pi}{2} - 1.34\right)$$
$$t = \left(\frac{\pi}{2} - 1.34\right) \div \frac{\pi}{6}$$
$$= 0.44 \ldots$$

Therefore, December has the longest day of the year.

1 mark – correct value for t

2 marks – correct translation of t into months.

(Students may use calculus, and they might start with $\sin(\frac{\pi t}{6} + 1.34) = 1$, either approach seems valid)

(c) A scientist needs 13 hours of continuous daylight for an experiment. Between which dates should she be looking to run her experiment? **3**

Let $y = 13$

$$13 = 2.24\sin\left(\frac{\pi t}{6} + 1.34\right) + 12.19$$
$$0.81 = 2.24\sin\left(\frac{\pi t}{6} + 1.34\right)$$
$$\frac{81}{224} = \sin\left(\frac{\pi t}{6} + 1.34\right)$$
$$\sin^{-1}\left(\frac{81}{224}\right) = \frac{\pi t}{6} + 1.34$$
$$\frac{\pi t}{6} + 1.34 = \sin^{-1}\left(\frac{81}{224}\right), \pi - \sin^{-1}\left(\frac{81}{224}\right), 2\pi + \sin^{-1}\left(\frac{81}{224}\right)$$
$$t = \frac{\sin^{-1}\left(\frac{81}{224}\right) - 1.34}{\frac{\pi}{6}}, \frac{\pi - \sin^{-1}\left(\frac{81}{224}\right) - 1.34}{\frac{\pi}{6}}, \frac{2\pi + \sin^{-1}\left(\frac{81}{224}\right) - 1.34}{\frac{\pi}{6}}$$
$$t = -1.817\ldots, 2.73\ldots, 10.14742\ldots$$

February has either 28/29 days
$0.73 \times 28 = 20.44$ or $0.69 \times 27 = 19.71$

October has 31 days
0.14*31=4.34

Therefore, the scientist should perform her experiment between October 5th and February 19th or 20th.

1 mark - finds one correct solutions, or translates two incorrect solutions into correct dates
2 marks – finds two correct solutions and attempts, or finds one correct solution and correctly translates to a date.
3 marks – finds two correct solutions and translates to the dates, specifying between October and February, rather than between February and October.

Markers feedback
These questions were very rarely seriously attempted. Part (a) and (b) were generally well done when attempted. Many students forgot that they would need to use radians, getting answers that were far too large. Some students found the correct dates, but set the region to be between February and October, rather than between October and February.

Question 37 (6 marks)

Do NOT write in this area.

Consider the function $f(x) = (x-38)e^{k-x}$, where k is a constant.

(a) By differentiating $f(x)$, find $\int (x-39)e^{k-x}dx$ **2**

$f'(x) = e^{k-x} - (x-38)e^{k-x}$ 1 mark for the derivative.
$= e^{k-x}(1+38-x)$
$= e^{k-x}(39-x)$ 1 mark for the correct integral.

$$\int (x-39)e^{k-x}dx$$
$$= -(x-38)e^{k-x}$$

(b) A chocolate bar is being manufactured by a machine which produces bars weighing between 39 g and 44 g. The weight of a bar produced can be modelled using the continuous random variable X which has a probability density function: **2**

$$p(x) = \begin{cases} (x-39)e^{k-x}, & \text{for } 39 \le x \le 44 \\ 0, & \text{for all other } x \text{ values} \end{cases}$$

Show that $k = 44 - \ln(e^5 - 6)$.

$\int_{39}^{44} (x-39)e^{k-x}dx = 1$ 1 mark for correctly setting up and substituting into the integral.
$1 = [(38-x)e^{k-x}]_{39}^{44}$
$1 = -6e^{k-44} + e^{k-39}$ 1 mark showing $k = 44 - \ln(e^5 - 6)$.
$1 = e^{k-44}(-6+e^5)$
$(e^5-6)^{-1} = e^{k-44}$
$\ln(e^5-6)^{-1} = k-44$
$k = 44 - \ln(e^5-6)$

(c) It is known that $P(39 \le X \le 40) < 0.3$. Show that the median weight of the chocolate bar produced is between 40 and 41 grams. **2**

$\int_{39}^{41} (x-39)e^{k-x}dx$ 1 mark for integrating between 39 and 41, or 40 and 41.
$= [(38-x)e^{k-x}]_{39}^{41}$
$= -2e^{3-\ln(e^5-6)} + 1e^{5-\ln(e^5-6)}$ 1 mark for a correct justifications with calculations.
$= 0.76 \ldots$
$\therefore as\ P(39 \le X \le 40) < 0.5$
and $P(39 \le X \le 41) > 0.5$
The median must be between 40 and 41.

Feedback

(a) This part was done reasonably well. Many students differentiated correctly, however, students who did not factorise the derivative struggled with the integral.

(b) This was done okay for those who has done part (a) correctly, but was very difficult for those who hadn't. Students needed some fluent skills with exponentials and logs for this one, and students should make sure they are comfortable with this.

(c) As this was the last question of the exam, very few students got 2 marks here. A common mistake was trying to solve $\int_{39}^{m}(x-39)e^{k-x}dx=\frac{1}{2}$, which did not work well for those who tried.

Students needed to show that $\int_{39}^{41}(x-39)e^{k-x}dx>0.5$, and justify why that means the median is between 40 and 41.

Students who integrated between 40 and 41 and used this as the reasoning did not get 2 marks unless they also calculated $P(39\leq X\leq 40)$ to go with it.

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Section II extra writing space

If you use this space, clearly indicate which question you are answering.

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